

Evaluate  $\int \frac{(\ln x)^2}{x^3} dx = \underbrace{-\frac{1}{2} x^{-2} (\ln x)^2}_{\textcircled{1}} - \underbrace{\frac{1}{2} x^{-2} \ln x}_{\textcircled{2}} - \underbrace{\frac{1}{4} x^{-2}}_{\textcircled{1}} + C$

SCORE: \_\_\_\_ / 5 PTS

MINUS  $\textcircled{\frac{1}{2}}$   
IF YOU FORGOT

$$\begin{array}{l} \frac{u}{(ln x)^2} + \frac{dv}{x^{-3}} \\ \frac{2 \ln x}{x} \quad - \frac{1}{2} x^{-2} \\ *x - \frac{2 \ln x}{x} - \frac{1}{2} x^{-3} * \frac{1}{x} \\ \frac{2}{x} \quad - \frac{1}{4} x^{-2} \\ *x - \frac{2}{x} - \frac{1}{4} x^{-3} * \frac{1}{x} \\ 0 \quad + \frac{1}{4} x^{-3} \\ \quad - \frac{1}{8} x^{-2} \end{array}$$

Prove the reduction formula  $\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$  (where  $n \neq 0$ ).

SCORE: \_\_\_\_ / 6 PTS

**NOTE: You must show how to get this formula.**

**You will receive 0 credit if your "proof" is differentiating both sides of the equation.**

$$\begin{aligned} & \begin{array}{l} \frac{u}{\cos^{n-1} u} \\ \frac{dv}{+ \cos u} \\ (n-1) \cos^{n-2} u \sin u \end{array} \begin{array}{l} \frac{dv}{\sin u} \end{array} \\ & \int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \sin^2 u \, du \\ & = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u (1 - \cos^2 u) \, du \\ & = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \, du - (n-1) \int \cos^n u \, du \\ & \left(\frac{1}{2}\right) n \int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \, du \\ & \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du \end{aligned}$$



Evaluate  $\int \frac{1}{x^2 \sqrt{4x^2 - 9}} dx$ .

$$4x^2 - 9 = 9(\sec^2 \theta - 1) = 9 \tan^2 \theta$$

$$4x^2 = 9 \sec^2 \theta \rightarrow \textcircled{1} x = \frac{3}{2} \sec \theta \rightarrow \sec \theta = \frac{2x}{3}$$

$$dx = \frac{3}{2} \sec \theta \tan \theta d\theta$$

SCORE: \_\_\_\_\_ / 6 PTS

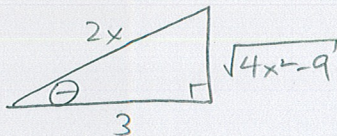
$$= \int \frac{1}{\frac{9}{4} \sec^2 \theta \cdot 3 \tan \theta} \cdot \frac{3}{2} \sec \theta \tan \theta d\theta \quad \textcircled{1}$$

$$= \frac{4}{9} \cdot \frac{1}{3} \cdot \frac{3}{2} \int \cos \theta d\theta \quad \textcircled{\frac{1}{2}}$$

$$= \frac{2}{9} \sin \theta + C \quad \textcircled{\frac{1}{2}}$$

$$= \frac{2}{9} \frac{\sqrt{4x^2 - 9}}{2x} + C$$

$$= \frac{\sqrt{4x^2 - 9}}{9x} + C \leftarrow \text{MINUS } \textcircled{\frac{1}{2}} \text{ IF YOU FORGOT}$$



Evaluate  $\int \arccos 2x \, dx = x \arccos 2x + \int \frac{2x}{\sqrt{1-4x^2}} \, dx$  (1 1/2)

SCORE: \_\_\_\_\_ / 5 PTS

$$\begin{array}{l} \underline{u} \quad \underline{dv} \\ \arccos 2x \quad + 1 \\ - \frac{2}{\sqrt{1-4x^2}} \quad = \frac{1}{x} \end{array}$$

$$\begin{array}{l} \uparrow \\ \underline{u = 1-4x^2} \rightarrow du = -8x \, dx \\ \textcircled{1} \quad - \frac{1}{4} du = 2x \, dx \end{array}$$

$$= x \arccos 2x - \frac{1}{2} \sqrt{1-4x^2} + C$$
(1 1/2)

MINUS (1/2)  
 IF YOU FORGOT

$$\begin{array}{l} \textcircled{1} \int -\frac{1}{4} \frac{1}{\sqrt{u}} \, du \\ = -\frac{1}{4} 2u^{\frac{1}{2}} \\ = -\frac{1}{2} u^{\frac{1}{2}} \\ = -\frac{1}{2} \sqrt{1-4x^2} \end{array}$$



Evaluate  $\int \sec^6 x \tan^6 x dx$ .

SCORE: \_\_\_\_ / 4 PTS

①  $u = \tan x$   $\rightarrow du = \sec^2 x dx$

$$\int \sec^4 x \tan^6 x \sec^2 x dx = \int (u^2 + 1)^2 u^6 du \quad \text{①}$$

$$= \int (u^{10} + 2u^8 + u^6) du$$

$$\text{① } \underline{\frac{1}{11} u^{11} + \frac{2}{9} u^9 + \frac{1}{7} u^7} + C$$

$$\text{① } \underline{\frac{1}{11} \tan^{11} x + \frac{2}{9} \tan^9 x + \frac{1}{7} \tan^7 x} + C$$

MINUS  $\left(\frac{1}{2}\right)$   
IF YOU FORGOT



Evaluate  $\int (x^2 - 3x - 4) \sin \frac{x}{2} dx = \underline{(x^2 - 3x - 4)(-2 \cos \frac{x}{2})}$ , ①

SCORE: \_\_\_\_ / 4 PTS

<u>u</u>		<u>dv</u>
$x^2 - 3x - 4$	+	$\sin \frac{x}{2}$
$2x - 3$	-	$-2 \cos \frac{x}{2}$
$2$	+	$-4 \sin \frac{x}{2}$
$0$	+	$8 \cos \frac{x}{2}$

+  $\underline{(2x - 3)(4 \sin \frac{x}{2})}$ , ①

① +  $\underline{2(8 \cos \frac{x}{2})} + C$

=  $\underline{(-2x^2 + 6x + 24) \cos \frac{x}{2} + (8x - 12) \sin \frac{x}{2} + C}$

①

↑  
MINUS  $(\frac{1}{2})$

IF YOU FORGOT